Solarthermal Power Plant (STPP) Technology

TEMO-STPP Test Stand 1: Numerical Simulations of Fluid Flow and Heat Transfer and Stress Analysis

Last update: 03 May 2010

- **Stress analysis in the heater with the Finite Elements (FEM) tool ABAQUS**
- **Modeling of the fluid flow at the border of the absorption pipes based on Navier-Stokes equations**

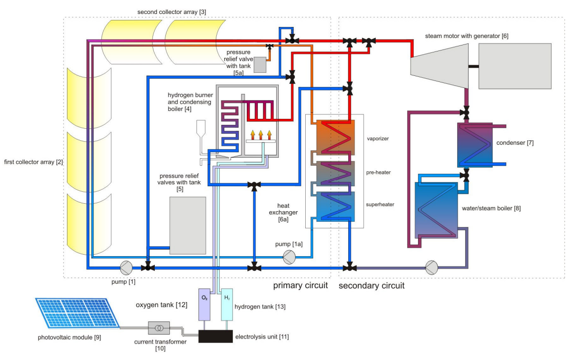

Fig.1: Principle of hybrid test stand

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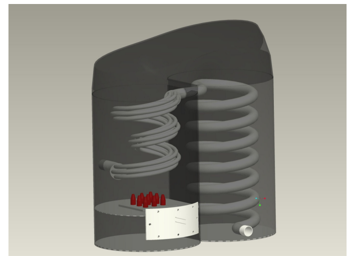
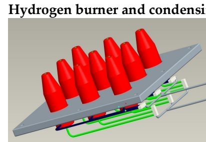
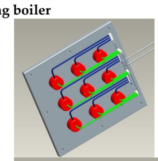
Fig. 2: The Heater

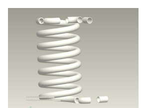
a mixture of 50 percentages hydrogen, domestic gas and 20-30 percentages methane as a catalyst (was in use in the newly-formed German states until 1996, known as "Stadtgas)

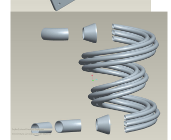


Hydrogen burner and condensing boiler









Exact calculations of the combusting process have to be done. Computational Fluid Dynamics (CFD) software, which allows displaying combusting processes is used to gain in experience in the air currents.

To garant a safe work a stress analysis of the burner is done with the FEM tool ABAQUS

The Absorption Pipe

The absorption pipe is one of the basic essentials of the system. Here, the thermal energy of the sun's rays is absorbed to be transmitted to the water/thermo oil. The absorption pipe is exactly located in the focal point of the parabolic mirrors, which are reflecting the sun. So the sun's rays are concentrated on the absorption pipe. Temperatures about 400°C are reached on the pipe's surface. That allows the heating of the water/thermo oil at least up to, which are required to run the steam engine.

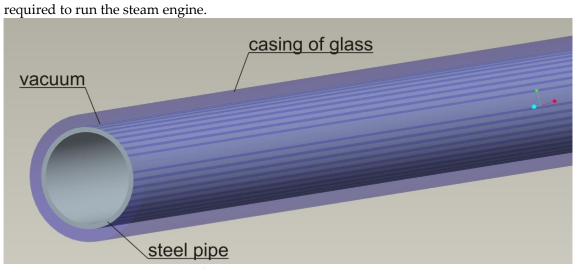


Figure: Sketch of the absorption pipe

The particular with the absorption pipe is a layer of vacuum inside. The active principle is the same as it is used for thermos bottles or double layer window pane. The absorption pipe consists of two interleaved pipes with this layer of vacuum inside to isolate thermal one pipe from the other.

In the ideal case vacuum is an absolute empty space. For thermal conduction in a space, at least the existence of atoms or gas molecules is necessary. So, there is no possibility for thermal conduction. For convective heat transfer any kind of stream is required. In vacuum stream does not take place because there is no material that could flow. So, there is also no possibility for convective heat transfer.

There is only heat radiation as a form of thermal transfer taking place in vacuum. The sun emits very strong heat radiation, which is amplified over again by concentrating the sun's rays by the parabolic mirrors. The high energy radiation shines through the external pipe, made of a special kind of glass, which has the same thermal expansion as steel has, to avoid tensions in the pipe. So the heat radiation gets to the internal pipe and heats up its surface. Through thermal conduction the heat is delivered to the fluid circulating in the pipe.

The effect of thermal conduction through the pipe can be increased by using special alloys.

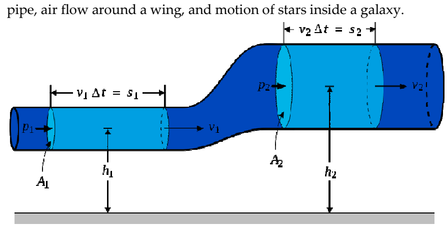
The vacuum around the internal pipe causes that the heat energy is not emitted to the environment but directly conducted in the pipe. The heated up pipes themselves also loose heat energy by heat radiation. But the intensity of this heat loss is, compared to heat of the concentrated sun rays, insignificant minor.

The absorption pipes will be delivered by an Austrian company. The diameter of these pipes is varying because of the pipes are to be optimized by being used in this project. That also effects advantages for the project and is welcome by the management of the TEMO STPP.

Modeling of the Fluid Flow at the border of the absortion pipes based on Navier-Stokes equations

The **Navier–Stokes equations**, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of fluid substances. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term.

The equations are useful because they describe the physics of many things of academic and economic interest. They may be used to model the weather, ocean currents, water flow in a pipe, air flow around a wing, and motion of stars inside a galaxy.



The Navier–Stokes equations dictate not position but rather velocity. A solution of the Navier–Stokes equations is called a velocity field or flow field, which is a description of the velocity of the fluid at a given point in space and time. Once the velocity field is solved for, other quantities of interest (such as flow rate or drag force) may be found. This is different from what one normally sees in classical mechanics, where solutions are typically trajectories of position of a particle or deflection of a continuum. Studying velocity instead of position makes more sense for a fluid, however for visualization purposes one can compute various trajectories.

The derivation of the Navier–Stokes equations begins with an application of Newton's second law: conservation of momentum (often alongside mass and energy conservation) being written for an arbitrary portion of the fluid. In an inertial frame of reference, the general form of the equations of fluid motion is:[2]

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \nabla\cdot\mathbb{T} + \mathbf{f},$$

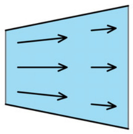
where $\mathbf{v}$ is the flow velocity, ρ is the fluid density, p is the pressure, $\mathbb{T}$ is the (deviatoric) stress tensor, and $\mathbf{f}$ represents body forces (per unit volume) acting on the fluid and ∇ is the del operator. This is a statement of the conservation of momentum in a fluid and it is an application of Newton's second law to a Continuum; in fact this equation is applicable to any non-relativistic continuum and is known as the Cauchy momentum equation.

This equation is often written using the substantive derivative $D\mathbf{v}/Dt$, making it more apparent that this is a statement of Newton's second law:

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla\cdot\mathbb{T} + \mathbf{f}.$$

The left side of the equation describes acceleration, and may be composed of time dependent or convective effects (also the effects of non-inertial coordinates if present). The right side of the equation is in effect a summation of body forces (such as gravity) and divergence of stress (pressure and shear stress).

Convective acceleration



An example of convection. Though the flow may be steady (time independent), the fluid decelerates as it moves down the diverging duct (assuming incompressible flow), hence there is an acceleration happening over position.

A very significant feature of the Navier–Stokes equations is the presence of convective acceleration: the effect of time independent acceleration of a fluid with respect to space. While individual fluid particles are indeed experiencing time dependent acceleration, the convective acceleration of the flow field is a spatial effect for example due to fluid speeding up in a nozzle. Convective acceleration is represented by the nonlinear quantity:

$$\mathbf{v}\cdot\nabla\mathbf{v},$$

which may be interpreted either as $(\mathbf{v}\cdot\nabla)\mathbf{v}$ or as $\mathbf{v}\cdot(\nabla\mathbf{v})$, with $\nabla\mathbf{v}$ the tensor derivative of the velocity vector $\mathbf{v}$. Both interpretations give the same result, independent of the coordinate system — provided ∇ is interpreted as the covariant derivative.[3]

Interpretation as (v·∇)v

The convection term is often written as

$$(\mathbf{v}\cdot\nabla)\mathbf{v},$$

where the advection operator $\mathbf{v}\cdot\nabla$ is used. Usually this representation is preferred because it is simpler than the one in terms of the tensor derivative $\nabla\mathbf{v}$.[3]

Interpretation as v·(∇v)

Here $\nabla\mathbf{v}$ is the tensor derivative of the velocity vector, equal in Cartesian coordinates to the component by component gradient. The convection term may, by a vector calculus identity, be expressed without a tensor derivative:[4][5]

$$\mathbf{v}\cdot\nabla\mathbf{v}=\nabla\left(\frac{\|\mathbf{v}\|^2}{2}\right)+(\nabla\times\mathbf{v})\times\mathbf{v}.$$

The form has use in irrotational flow, where the curl of the velocity (called vorticity) $\omega=\nabla\times\mathbf{v}$ is equal to zero.

Regardless of what kind of fluid is being dealt with, convective acceleration is a nonlinear effect. Convective acceleration is present in most flows (exceptions include one-dimensional incompressible flow), but its dynamic effect is disregarded in creeping flow (also called Stokes flow) .

Stresses

The effect of stress in the fluid is represented by the ∇p and $\nabla\cdot\mathbb{T}$ terms, these are gradients of surface forces, analogous to stresses in a solid. ∇p is called the pressure gradient and arises from the isotropic part of the stress tensor. This part is given by normal stresses that turn up in almost all situations, dynamic or not. The anisotropic part of the stress tensor gives rise to $\nabla\cdot\mathbb{T}$, which conventionally describes viscous forces; for incompressible flow, this is only a shear effect. Thus, $\mathbb{T}$ is the deviatoric stress tensor, and the stress tensor is equal to:[6]

$$\sigma=-p\mathbb{I}+\mathbb{T}$$

where $\mathbb{I}$ is the 3×3 identity matrix. Interestingly, only the *gradient* of pressure matters, not the pressure itself. The effect of the pressure gradient is that fluid flows from high pressure to low pressure.

The stress terms p and $\mathbb{T}$ are yet unknown, so the general form of the equations of motion is not usable to solve problems. Besides the equations of motion—Newton's second law—a force model is needed relating the stresses to the fluid motion.[7] For this reason, assumptions on the specific behavior of a fluid are made (based on natural observations) and applied in order to specify the stresses in terms of the other flow variables, such as velocity and density.

The Navier–Stokes equations result from the following assumptions on the deviatoric stress tensor $\mathbb{T}$:[8]

- the deviatoric stress vanishes for a fluid at rest, and – by Galilean invariance – also does not depend directly on the flow velocity itself, but only on spatial derivatives of the flow velocity
- in the Navier–Stokes equations, the deviatoric stress is expressed as the product of the tensor gradient $\nabla\mathbf{v}$ of the flow velocity with a viscosity tensor $\mathbb{A}$, i.e. : $\mathbb{T}=\mathbb{A}(\nabla\mathbf{v})$
- the fluid is assumed to be isotropic, as valid for gases and simple liquids, and consequently $\mathbb{A}$ is an isotropic tensor; furthermore, since the deviatoric stress tensor is symmetric, it turns out that it can be expressed in terms of two scalar dynamic viscosities μ and μ'': $\mathbb{T}=2\mu\mathbb{E}+\mu''\Delta\mathbb{I}$, where $\mathbb{E}=\frac{1}{2}(\nabla\mathbf{v})+\frac{1}{2}(\nabla\mathbf{v})^{\mathrm{T}}$ is the rate-of-strain tensor and $\Delta=\nabla\cdot\mathbf{v}$ is the rate of expansion of the flow
- the deviatoric stress tensor has zero trace, so for a three-dimensional flow $2\mu+3\mu''=0$

As a result, in the Navier–Stokes equations the deviatoric stress tensor has the following form:[8]

$$\mathbb{T}=2\mu\left(\mathbb{E}-\tfrac{1}{3}\Delta\mathbb{I}\right),$$

with the quantity between brackets the non-isotropic part of the rate-of-strain tensor $\mathbb{E}$. The dynamic viscosity μ does not need to be constant – in general it depends on conditions like temperature and pressure, and in turbulence modelling the concept of eddy viscosity is used to approximate the average deviatoric stress.

The pressure p is modelled by use of an equation of state.[9] For the special case of an incompressible flow, the pressure constrains the flow in such a way that the volume of fluid elements is constant: isochoric flow resulting in a solenoidal velocity field with $\nabla\cdot\mathbf{v}=0$.[10]

Other forces

The vector field $\mathbf{f}$ represents "other" (body force) forces. Typically this is only gravity, but may include other fields (such as electromagnetic). In a non-inertial coordinate system, other "forces" such as that associated with rotating coordinates may be inserted.

Often, these forces may be represented as the gradient of some scalar quantity. Gravity in the z direction, for example, is the gradient of $-\rho g z$. Since pressure shows up only as a gradient, this implies that solving a problem without any such body force can be mended to include the body force by modifying pressure.

Other equations

The Navier–Stokes equations are strictly a statement of the conservation of momentum. In order to fully describe fluid flow, more information is needed (how much depends on the assumptions made), this may include boundary data (no-slip, capillary surface, etc), the conservation of mass, the conservation of energy, and/or an equation of state.

Regardless of the flow assumptions, a statement of the conservation of mass is generally necessary. This is achieved through the mass continuity equation, given in its most general form as:

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$$

or, using the substantive derivative:

$$\frac{D\rho}{Dt}+\rho(\nabla\cdot\mathbf{v})=0.$$

Incompressible flow of Newtonian fluids

A simplification of the resulting flow equations is obtained when considering an incompressible flow of a Newtonian fluid. The assumption of incompressibility rules out the possibility of sound or shock waves to occur; so this simplification is invalid if these phenomena are important. The incompressible flow assumption typically holds well even when dealing with a "compressible" fluid — such as air at room temperature — at low Mach numbers (even when flowing up to about Mach 0.3). Taking the incompressible flow assumption into account and assuming constant viscosity, the Navier–Stokes equations will read, in vector form:[11]

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\mathbf{f}.$$

Here **f** represents "other" body forces (forces per unit volume), such as gravity or centrifugal force. The shear stress term $\nabla\cdot\mathbb{T}$ becomes the useful quantity $\mu\nabla^2\mathbf{v}$ when the fluid is assumed incompressible and Newtonian, where μ is the dynamic viscosity.[12]

It's well worth observing the meaning of each term (compare to the Cauchy momentum equation):

$$\overbrace{\rho\Big(\underbrace{\frac{\partial\mathbf{v}}{\partial t}}_{\begin{smallmatrix}\text{Unsteady}\\\text{acceleration}\end{smallmatrix}}+\underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\begin{smallmatrix}\text{Convective}\\\text{acceleration}\end{smallmatrix}}\Big)}^{\text{Inertia (per volume)}}=\overbrace{\underbrace{-\nabla p}_{\begin{smallmatrix}\text{Pressure}\\\text{gradient}\end{smallmatrix}}+\underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}}}^{\text{Divergence of stress}}+\underbrace{\mathbf{f}}_{\begin{smallmatrix}\text{Other}\\\text{body}\\\text{forces}\end{smallmatrix}}.$$

Note that only the convective terms are nonlinear for incompressible Newtonian flow. The convective acceleration is an acceleration caused by a (possibly steady) change in velocity over *position*, for example the speeding up of fluid entering a converging nozzle. Though individual fluid particles are being accelerated and thus are under unsteady motion, the flow field (a velocity distribution) will not necessarily be time dependent.

Another important observation is that the viscosity is represented by the vector Laplacian of the velocity field (interpreted here as the difference between the velocity at a point and the mean velocity in a small volume around). This implies that Newtonian viscosity is **diffusion of momentum**, this works in much the same way as the diffusion of heat seen in the heat equation (which also involves the Laplacian).

If temperature effects are also neglected, the only "other" equation (apart from initial/boundary conditions) needed is the mass continuity equation. Under the incompressible assumption, density is a constant and it follows that the equation will simplify to:

$$\nabla\cdot\mathbf{v}=0.$$

This is more specifically a statement of the conservation of volume (see divergence).

These equations are commonly used in 3 coordinates systems: Cartesian, cylindrical, and spherical. While the Cartesian equations seem to follow directly from the vector equation above, the vector form of the Navier–Stokes equation involves some tensor calculus which means that writing it in other coordinate systems is not as simple as doing so for scalar equations (such as the heat equation).

Cartesian coordinates

Writing the vector equation explicitly,

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)+\rho g_x$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)+\rho g_y$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+\rho g_z.$$

Note that gravity has been accounted for as a body force, and the values of g_x, g_y, g_z will depend on the orientation of gravity with respect to the chosen set of coordinates.

The continuity equation reads:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0.$$

When the flow is at steady-state, ρ does not change with respect to time. The continuity equation is reduced to:

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0.$$

When the flow is incompressible, ρ is constant and does not change with respect to space. The continuity equation is reduced to:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0.$$

The velocity components (the dependent variables to be solved for) are typically named u, v, w. This system of four equations comprises the most commonly used and studied form. Though comparatively more compact than other representations, this is still a nonlinear system of partial differential equations for which solutions are difficult to obtain.